Assignment 9.

This homework is due *Tuesday* 11/30/2010.

There are total of 51 points in this assignment. 44 points is considered 100%. If you go over 44 points, you will get over 100% for this homework and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper.

This assignment covers sections 6.4 and beginning of 7.1.

(1) (a) [3pt] For the function $f(x) = \sin x$, find the following Taylor's polynomials at $x_0 = 0$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_5(x)$, $P_{2010}(x)$.

(b) [3pt] For the function $f(x) = \sin x$, find the following Taylor's polynomials at $x_0 = \pi/2$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_5(x)$, $P_{2010}(x)$.

(c) [3pt] For the function $f(x) = \cos x$, find the following Taylor's polynomials at $x_0 = 0$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_5(x)$, $P_{2010}(x)$. Compare to the previous item.

(d) [3pt] For the function $f(x) = \cos x$, find the following Taylor's polynomials at $x_0 = \pi$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_5(x)$, $P_{2010}(x)$.

(e) [3pt] For the function $f(x) = x^3$, find the following Taylor's polynomials at $x_0 = 2$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_{2010}(x)$. Find the remainder $R_3(x)$. Compare $P_3(x), P_4(x), P_{2010}(x)$ to $f(x)$.

(f) [3pt] For the function $f(x) = \frac{1}{1-x}$, find the following Taylor's polynomials at $x_0 = 0$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_{2010}(x)$.

(g) [3pt] For the function $f(x) = \frac{1}{x}$, find the following Taylor's polynomials at $x_0 = 1$: $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_{2010}(x)$. Compare to the previous item.

(2) [4pt] (Part of exercise 6.4.7) If $x > 0$, show that

$$\left| \sqrt[3]{1+x} - \left(1 + \frac{1}{3}x - \frac{1}{9}x^2\right) \right| \le \frac{5}{81}x^3.$$

(3) (a) [3pt] Suppose $A \in \mathbb{R}$. Show that $\lim\limits_{n\to\infty} \frac{A^n}{n!} = 0$ (Hint: take tail of this sequence that starts with $n_0 > 2|A|$.)

(b) [4pt] (Exercise 6.4.8) If $f(x) = e^x$, show that the remainder term in Taylor's Theorem converges to zero as $n \to \infty$, for each fixed x_0 and x.

(c) [4pt] (Exercise 6.4.9) If $g(x) = \sin x$, show that the remainder term in Taylor's Theorem converges to zero as $n \to \infty$, for each fixed x_0 and x.

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(4) (Part of exercise 6.4.14) Determine whether or not $x = 0$ is a point of relative extremum of the following functions:
  (a) [2pt] $f(x) = x^3 + 2$,
  (b) [3pt] $f(x) = \sin x - x$,
  (c) [3pt] $f(x) = \cos x - 1 + \frac{1}{2}x^2$.

(5) (Part of exercise 7.1.1) If $I = [0, 1]$, calculate the norms of the following partitions:
  (a) [1pt] $\mathcal{P}_1 = (0, 1, 2, 4)$,
  (b) [1pt] $\mathcal{P}_2 = (0, 2, 3, 4)$,
  (c) [1pt] $\mathcal{P}_3 = (0, 1, 1.5, 2, 3.4, 4)$.

(6) (Part of exercise 7.1.2) If $f(x) = x^2$ for $x \in [0, 4]$, calculate the following Riemann sums, where $\dot{\mathcal{P}}_i$ has the same partition points as in the previous problem, and the tags are selected as indicated.
  (a) [2pt] $\mathcal{P}_1$ with the tags at the left endpoints of the subintervals.
  (b) [2pt] $\mathcal{P}_2$ with the tags at the right endpoints of the subintervals.